U.G. 2nd Semester Examinations 2022

MATHEMATICS (General)

Paper Code : MTMG DC-2

[CBCS]

Full Marks : 32 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Group - A

(4 Marks)

1. Answer any ***four*** questions : 1×4=4

   (a) Find the limit $\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$.

   (b) Find the value of $B\left(\frac{1}{2},\frac{1}{2}\right)$, where B denotes the beta function.

   (c) State Cauchy's general principle of convergence for sequences of real numbers.

   (d) Find an integrating factor of the differential equation

   $$x\cos x\frac{dy}{dx}+y\left(x\sin x+\cos x\right)=1.$$

   (e) What do you mean by an absolutely convergent series ?

   (f) Solve the differential equation : $\log\left(\frac{dy}{dx}\right)=ax+by$.

   (g) Show that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=|x|$ for all $x\in\mathbb{R}$ is not differentiable at 0.

Group - B

(10 Marks)

Answer any *two* questions. 5×2=10

2. Evaluate $\lim_{n\to\infty}\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\cdots\left(1+\frac{n}{n}\right)\right\}^{\frac{1}{n}}$. 5

[P.T.O.]

3. Solve the differential equation : $(\sin y)\frac{dy}{dx} = \cos x\left(2\cos y - \sin^2 x\right)$. 5

4. Using the Mean Value Theorem, show that

$\frac{x}{1+x} < \log(1+x) < x$, where $x > 0$. 5

5. Examine the convergence of $\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx$. 5

Group - C

(18 Marks)

Answer any *two* questions. 9×2=18

6. (a) Find the value of $\int_0^{\pi/2} \sin^5\theta \cos^7\theta\, d\theta$. 6

(b) Evaluate $\lim_{x\to 0+}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$. 3

7. (a) If $y = x^{n-1}\log x, n \in \mathbb{N}$, prove that $y_n = \frac{(n-1)!}{x}$. 5

(b) Test the convergence of the series

$\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots$ 4

8. (a) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = e^x \cos x$. 5

(b) If $I_n = \int_0^{\pi/4} \tan^n x\, dx, n \in \mathbb{N}$, show that $I_n + I_{n-2} = \frac{1}{n-1}$. 4
